Array shape estimation method based on extraction of single normal mode

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ABSTRACT

Accurate estimation and knowledge of array shape is essential for array signal processing in underwater target localization. This paper describes a method to extract a single mode and estimate array shape based on warping transform with an impulse source. One can process the signal received by a vertical array and transform it to another in the wave guide invariant (β) form for warping transform, then extract the single normal mode. By constructing the relation between the vertical array acoustic field phase and vertical array shape, the array shape can be estimated by normal mode phase. There is no source localization information needed in this method, and the simulations in different underwater environment show that the vertical array shape can be estimated effectively.

1 INTRODUCTION

Due to the impact of ocean currents and the disturbance caused by deployment, the underwater long line array will seriously deform and bend, which seriously reduces the performance of underwater target detection. In order to better match the underwater array signals, it is necessary to know the array shape accurately. In order to achieve less than 1dB loss of signal gain, error of array elements position estimation must be less than $\lambda/10$ at the frequency of interest.

There are three categories of array shape estimation methods. By placing inclinometers and compasses in the line array, senor method can obtain the array shape by real-time monitoring the sensor values. Acoustic method uses the array received acoustic signals to invert the array shape, such as high-frequency (Jonathan L. Odom, 2015), matched-field (William S. Hodgkiss, 1996), environment noise. Hydrodynamic modal method is based on fluid stress response of the line array system through modal calculation. The acoustic method can correct the array shape through active sound source, and also can realize the autonomous positioning of the line elements by environmental noise (K. G. Sabra, 2005).

Based on an ideal waveguide with perfect reflection boundaries, warping transform has been introduced to compensate for the waveguide dispersive effect and isolate the modal components (G. Le Touzé, 2009; J. Bonnel and N. Chapman, 2011). There is also other warping transform for special sound speed profile, such as a linearly increased sound speed profile (Qi Yu-Bo, 2016). Based on the waveguide invariant (β), which summarizes the dispersive characteristics of the field in a waveguide in a single scalar parameter, a general warping transform is presented. Modal depth function can be estimated using time-frequency analysis of warping transform (J. Bonnel and C. Gervaise, 2011). Inspired by the modal depth function estimation, this paper employs the β-warping transform to obtain the phase of signal, hence to estimate the array shape.

Compared with the method of acoustic time delay difference for array shape correction, the vertical line array shape is obtained by estimating the phase information of the received sound field in this paper, hence a better result is achieved. The matched field method also makes use of phase information of the sound field; however, it is vulnerable to environmental mismatch and the impact of copy field calculation. This method does not need to calculate the replica field and less restricted by environmental factors, hence is more robust.

The remaining parts of the paper are organized as follows. Theoretical derivation is presented in Section 2, single normal mode field is extracted from the pulse signal, and the phase information of the signals is obtained. The relation of the vertical array element horizontal offset and the phase information is given. The vertical array shape estimation under different environment is simulated in Section 3. Conclusions are drawn in Section 4.

2 ARRAY SHAPE ESTIMATION METHOD

The source and vertical line array are assumed in a vertical plane, and the position error of vertical array within this plane is considered in this paper, three-dimensional incline of the array is out of the topic of this paper. Also, the source is assumed in an azimuthally symmetric environment.

2.1 Normal mode extraction

According to normal mode theory, the sound pressure field of a point source in layered shallow water can be expressed in sum of a serial of normal modes. For a pulse of source, the received wave form is the integral of the source frequency spectrum and the point source field. The vertical array element receiving the signal at depth(z) and range(r) in far field can be express as:

$$p(t,r,z) = \int S(\omega)P(\omega,r,z)\exp(-j\omega t)d\omega = A\sum_m \int \left(\frac{S(\omega)\psi_m(\omega,z_s)\psi_m(\omega,z)\exp[j(k_m(\omega)r-\omega t]}{\sqrt{k_m(\omega)}}\right)d\omega \tag{1}$$

Where $k_m$and ψ_m are the number m of normal mode eigenvalue and eigen function, $A = exp(j\pi/4)/\left(\rho(z_s)\sqrt{8\pi r}\right)$. If the sound frequency spectrum is a slowly varying function, only the exponential term factor is considered, and the sound field is obtained by the stationary phase approximation (L. M. Brekhovskikh, 1992), as follow:

$$p(t,r,z) = \sum B_m(\omega_0)\exp[j(k_m(\omega_0)r - \omega_0 t)] \tag{2}$$

In which $B_m(\omega_0)$ and $k'_m(\omega_0)$ are

$$B_m(\omega_0) = AS(\omega_0)\psi_m(\omega_0,z_s)\psi_m(\omega_0,z)\sqrt{\frac{2\pi}{rk_m(\omega_0)|k''_m(\omega_0)|}}\exp(\pm j\pi/4) \tag{3}$$

$$k'_m(\omega_0) = \frac{t}{r} \tag{4}$$

$k''_m(\omega_0)$ is the second derivative of $k_m(\omega)$, $\pm$ is the sign of $k''_m(\omega_0)$. Eq. (4) is derived from the stationary phase approximation condition, which satisfies equation$(k_m(\omega)r - \omega t)' = 0$. According to the definition of waveguide invariant: $1/\beta = -\partial S_g/\partial S_p$, $S_g$and S_p are group slowness and phase slowness respectively, combined with WKB approximate (G. L. D'Spain, 1999), the normal mode eigenvalue can be expressed approximate by:

$$k_m(\omega) = \frac{\omega}{c(z)} - \frac{C_2^m\omega^{-1/\beta}}{c(z)} \tag{5}$$

where C_2^m is a mode dependent constant, $c(z)$ is the depth dependent sound speed. Using Eqs. (4) and (5), the phase factor of the exponent of Eq. (2) can be written as:

$$\varphi_m = -\left(\beta^{-\frac{1}{1+\beta}} + \beta^{\frac{1}{1+\beta}}\right)\left(\frac{C_2^m r}{c(z)}\right)^{\frac{\beta}{1+\beta}}\left(t - \frac{r}{c(z)}\right)^{\frac{1}{1+\beta}} \tag{6}$$

The warping transform function is defined (Licheng Lu, 2015) as:

$$w(t) = t^{1+\beta} + \frac{r}{c(z)} \tag{7}$$

The warped time and frequency is related with original time and frequency as:

$$\tilde{t} = \left(t - \frac{r}{c(z)}\right)^{\frac{1}{1+\beta}} \tag{8}$$

$$\tilde{f} = (1+\beta)\left(t - \frac{r}{c(z)}\right)^{\frac{1}{1+\beta}} f \tag{9}$$

It is usually chosen $w(t)$ that the warped signal is linear in the time or frequency domain. The broadband signal obtained by warping transform in frequency domain is linear, so individual normal mode can be separated by filtering, and then the signal waveform of single mode is obtained by inverse warping transform of the filtered data. The warping transform based on the waveguide invariant (β) is different from the ideal waveguide warping transform, which is $w(t) = \sqrt{t^2 + t_r^2}$, $t_r = \frac{r}{c_w}$, r is the range between source and receiver and c_w is the sound speed of water. So this warping transform does not need to know the location of the source.

2.2 Relation of phase and array element offset

As mentioned above, single normal mode frequency field of vertical array can be obtained using Fourier transform from single normal mode waveform. When the vertical array is curved and the horizontal offset is Δr, then the vertical array sound field in a single mode is expressed as:

$$P_m(\omega, r, z_n) = S(\omega)\frac{\exp\left(\frac{j\pi}{4}\right)}{\rho(z_s)\sqrt{8\pi}}\psi_m(\omega, z_s)\psi_m(\omega, z_n)\frac{\exp[jk_m(\omega)(r + \Delta r)]}{\sqrt{k_m(\omega)(r + \Delta r)}}$$

$$= \left|\frac{S\psi_m(\omega,z_s)\psi_m(\omega,z_n)}{\rho(z_s)\sqrt{8\pi}\sqrt{k_m(\omega)(r+\Delta r)}}\right| \exp\left\{j\left[k_m(r + \Delta r) + \phi_s + \frac{\pi}{4} + \phi_z\right]\right\} \quad (10)$$

ϕ_s is the phase of $S(\omega)$ and ϕ_z is the phase of $\psi_m(\omega, z_n)$, n is the number of the array element. If the sign of $\psi_m(\omega, z_n)$ (which is real) is constant, then $\phi_z = 0$. If the sign of $\psi_m(\omega, z_n)$ changes at depth z_n, then the phase ϕ_z must jump by a factor π at depth z_n. $k_m r$ and ϕ_s is constant for the m mode. Therefore, the horizontal offset of the vertical array is linearly related to the phase expect for compensating π:

$$\Delta r_n = (\Phi_n - \Phi_{n-1})/k_m \approx (\Phi_n - \Phi_{n-1})/k_0 \quad (11)$$

where $\Phi_n = \arg\left(P_m(\omega, r, z_n)\right) - \mathrm{sign}(\psi_m(\omega, z_n)) \cdot \pi$, the wave number k_0 is obtained from the average sound velocity in sea water. The sign of $\psi_m(\omega, z_n)$ can be estimated following (J. Bonnel and C. Gervaise, 2011). For two consecutive elements *n* and *n+1* of the VLA, the quantity a_n is computed as:

$$a_n = mod\left\{\left[arg\left(P_m(\omega, r, z_{n+1})\right) - arg\left(P_m(\omega, r, z_n)\right)\right], 2\pi\right\} \quad (12)$$

Assuming that the horizontal offset difference $\Delta r_n - \Delta r_{n-1}$ of two consecutive elements is smaller than one fourth of the wavelength, if $a_n > \pi/2$, then $\psi_m(\omega, z_n)$ changes the sign. Once the horizontal offset Δr_n is obtained, the vertical offset $\Delta z_n$of two adjacent elements can be calculated from $\Delta z_n = \sqrt{L^2 - \Delta r_n^2}$, where L is the length of two adjacent elements.

3 SIMULATION

3.1 Simple environment

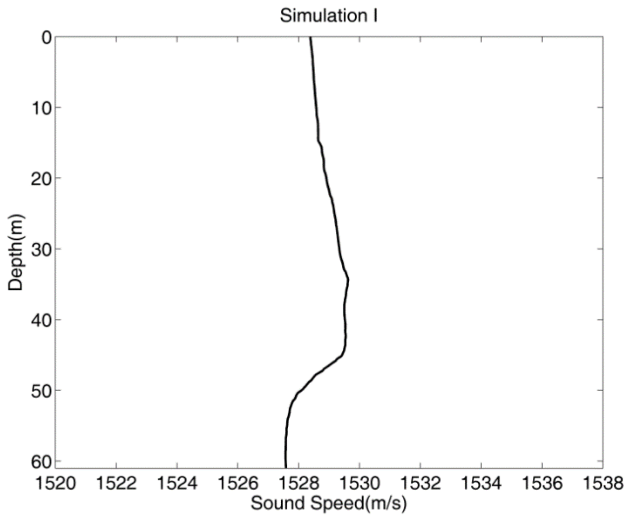

Figure 1: Sound speed profile for simple environment simulation

Sound velocity profile of approximate homogeneous water column is assumed in this numerical simulation, as shown in Fig. 1. The acoustic source is a gauss pulse signal with a centre frequency of 300Hz. The depth of the sound source is 15m, and the distance between sound source and vertical array is 15km. The 32 elements vertical array is deployed between 8m and 54m where the water depth is 61m. The seabed is semi-infinite with the following assumption: seabed velocity 1600m/s, density 1.6g/cm^3 and attenuation coefficient 0.2dB/λ. The array shape is assuming to be a quadratic curve and the tangent line of the top element is vertical. The top element is offset -5m toward the source, then after simple mathematic calculation, the array element location can be determined.

The received signal (Fig. 3a) of the 16th channel signal in the vertical array is used to carry out the warping transformation by Eqs. (7). Time-frequency analysis diagrams of the 16th channel are shown in Fig. 2. Fig. 2a and Fig. 2b show time-frequency analysis of the original signal and transform signal. After transformation, the mode signal in frequency domain is a continuous wave, and can be filtered (Fig. 2d). The filtered single mode waveform can be obtained by inverse transform (Fig. 2c).

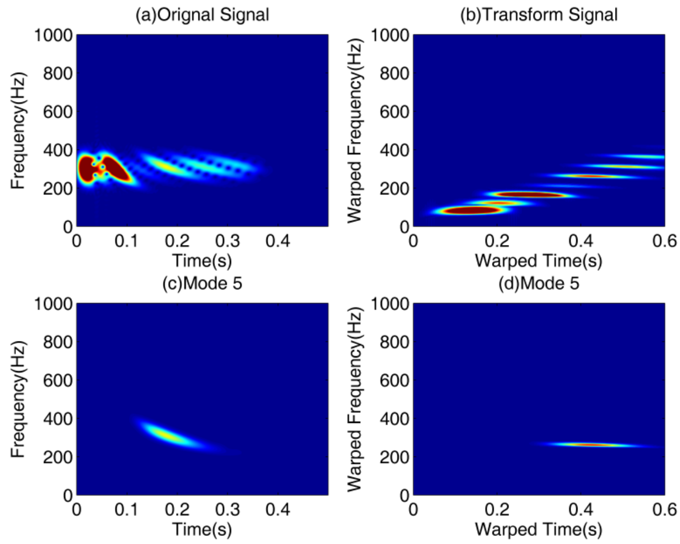

Figure 2: Time-frequency analysis of original and transformed signal (I)

Fig. 3 shows the waveform of the original signal and the mode 5 waveform. The single mode waveform is applied Fourier transform, and 250Hz, 300Hz and 350Hz are selected to calculate array element phase information. Then horizontal offset of vertical array can calculated using Eqs. (11). Fig. 4 shows the results of mode 1, mode 2 and mode 5 at these frequencies.

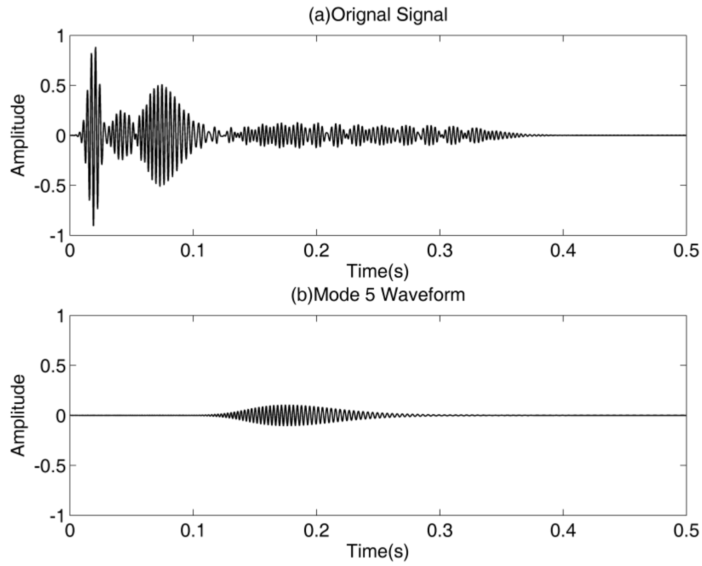

Figure 3: Original signal and single mode waveform (I, mode 5)

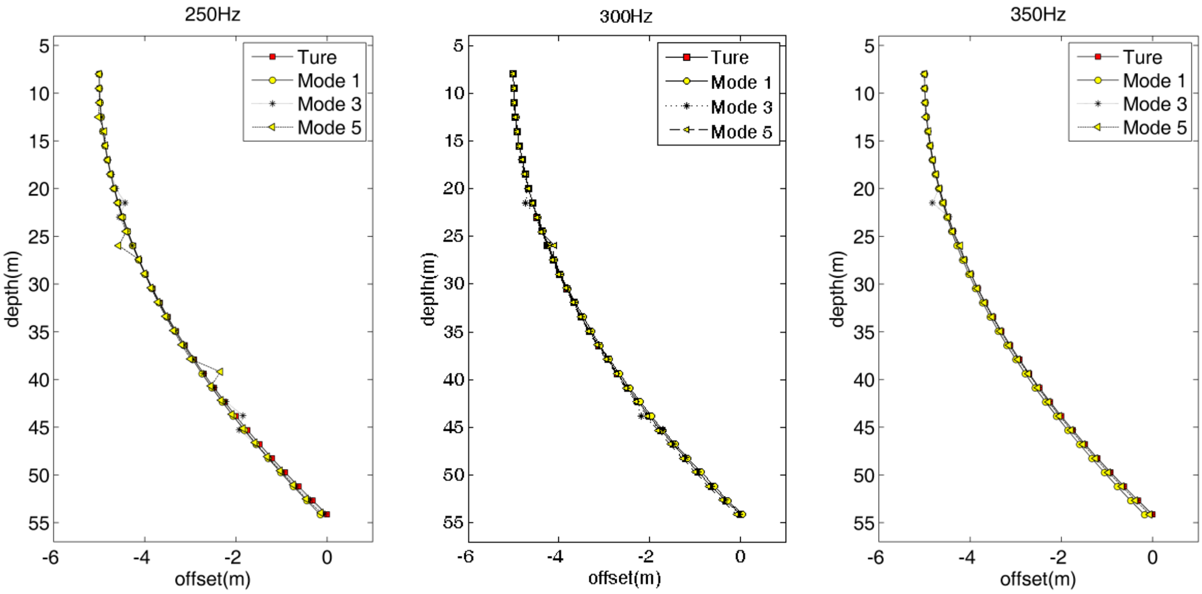

Figure 4: Comparison between the estimated array shape and true (I)

3.2 Complicate environment

Sound velocity profile of refracting water column is assumed in this numerical simulation, as shown in Fig. 5. The 32 elements vertical array is deployed between 15m and 60m where the water depth is 62m. A two layers seabed is assumed. Thickness of the top layer is 5m, with surface velocity of 1560m/s, bottom velocity of 1570m/s, density of 1.6g/cm3, and attenuation coefficient of 0.1dB/ λ , the parameters of basement are as follow: velocity 1650m/s, density 1.7g/cm^3, attenuation coefficient 0.1dB/λ. The top element is offset 10m away from the source. Other parameters are the same as with simulation in 3.1.

Using the same processing method as in 3.1, Fig. 6 shows a time-frequency analysis of the 16th channel signal. Fig. 7 shows the waveform of the original signal and the waveform of the mode 2. The single mode signal waveform is obtained by FFT decomposition, 250Hz, 300Hz and 350Hz are selected to calculate array element phase information. Fig. 8 shows the results of mode 2, mode 3 and mode 5 at these frequencies.

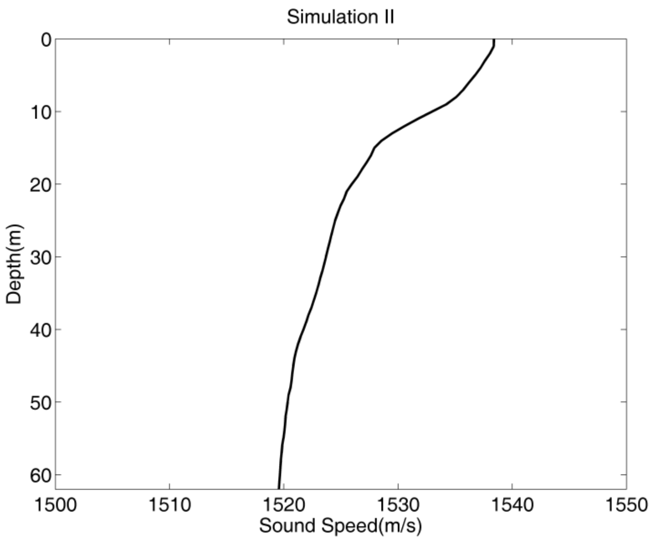

Figure 5: Sound speed profile for complicate environment simulation

Table I shows the errors between estimated array shape and the truth in different mode and different frequency for two simulations. The maximum errors of Δr_n and Δz_n are 0.771m and 0.686m which are calculated with mode 2 of 350Hz in simulation II. The reason may be for high frequency and low order mode, the single mode extraction becomes difficult. However, maximum RMS of Δr_n and Δz_n are small, 0.178m and 0.268m respectively.

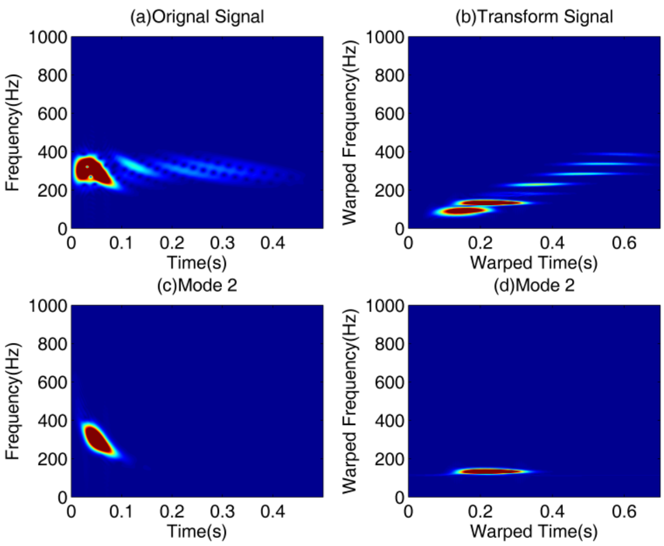

Figure 6: Time-frequency analysis of original and transformed signal (II)

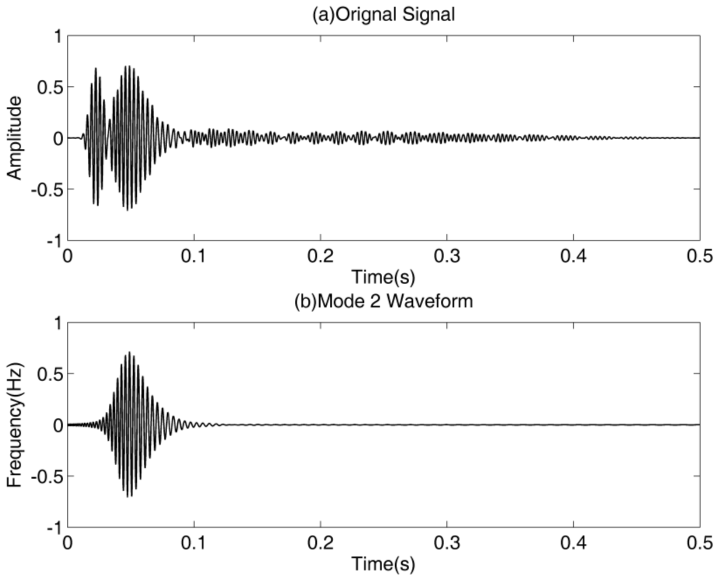

Figure 7: Original signal and single mode waveform (II, mode 2)

Table 1: Errors between the estimated array element position and true

	Simulation I									Simulation II								
Frequency(Hz)	250			300			350			250			300			350		
Mode	1	3	5	1	3	5	1	3	5	2	3	5	2	3	5	2	3	5
Maximum Δr_n(m)	0.147	0.018	0.365	0.060	0.170	0.163	0.178	0.250	0.060	0.416	0.193	0.539	0.273	0.113	0.267	0.771	0.164	0.197
Maximum Δz_n(m)	0.024	0.062	0.195	0.007	0.054	0.024	0.025	0.046	0.007	0.169	0.032	0.276	0.187	0.024	0.079	0.686	0.024	0.058
RMS Δr_n(m)	0.040	0.060	0.099	0.020	0.042	0.044	0.051	0.044	0.021	0.094	0.067	0.178	0.078	0.040	0.083	0.172	0.042	0.060
RMS Δz_n(m)	0.006	0.021	0.082	0.002	0.017	0.008	0.007	0.020	0.002	0.063	0.014	0.086	0.067	0.010	0.024	0.261	0.009	0.017

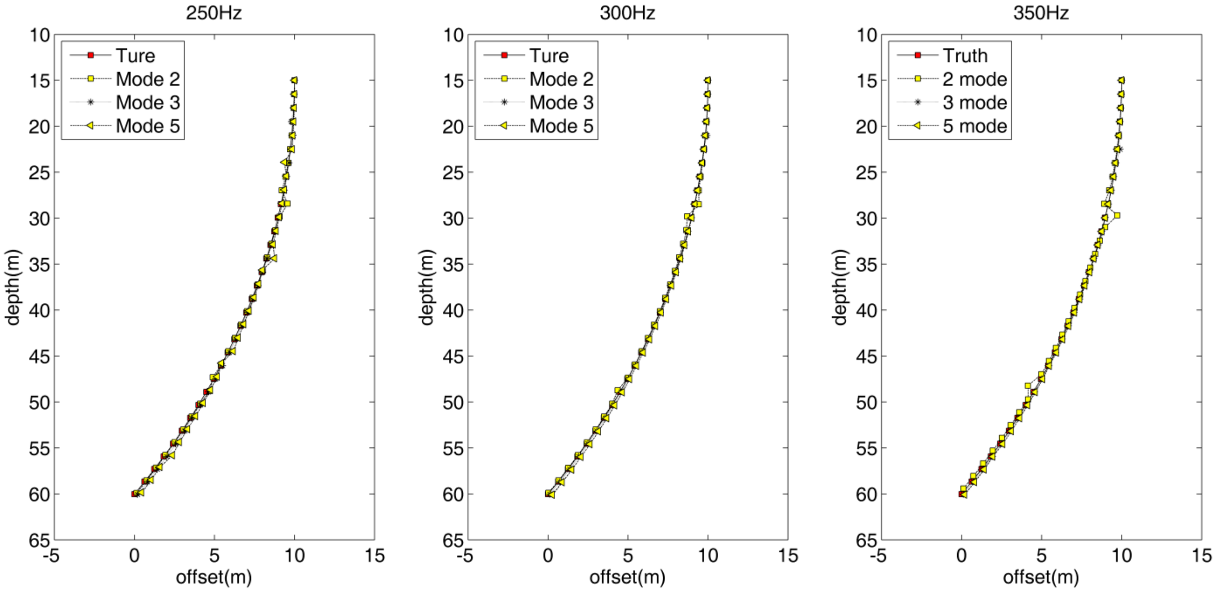

Figure 8: Comparison between the estimated array shape and true (II)

4 CONCLUSIONS

This paper employs warping transform to obtain pulse sound source single normal mode. The phase information of vertical array signals is used to estimate vertical array shape. Vertical array shape can be estimated using phase information of vertical array signals with no knowledge of source location. Numerical simulation are carried out in two typical environments (simple winter and complicate summer), and the result shows that vertical array shape can be estimated effectively when the maximum offset of the element is within 2 wavelengths. When the vertical element is located in the modal function nodes, it will affect the accuracy of phase estimation. The effect on phase extraction due to noise in real environment is under further investigation.

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