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Determination of the calibration error of a reciprocal underwater acoustic transducer from standard data obtained in a two-way comparison calibration process using the acoustic reciprocity parameter

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ABSTRACT

Hydrophones are more commonly calibrated using the two-way comparison method than the three-way reciprocity method. The comparison method is the most frequent chosen technique for hydrophone calibration due to the time-consuming nature of conducting the free field three-way reciprocity technique. This method is chosen despite the three-way reciprocity method resulting in a more accurate calibration than that of the comparison method due to it being an absolute method of calibration. This paper illustrates how to derive the indeterminate error in a two-way comparison calibration using the reciprocity parameter with the limited data supplied from a standard commercial calibration process if the transducer under test is reciprocal. The proposed simple method demonstrates that the determination of this error can be derived whilst examining the reciprocity of the transducer under test and can be performed post calibration using the supplied data consisting of complex impedance (Z) and the measured receive (M) and transmit (S) sensitivities of the transducer under test. This method is advantageous as one does not have to break the measuring circuit to observe other variables such as voltage and current at the transducer terminals to prove reciprocity which is difficult to do during an automated comparison calibration process. It is shown that from this proposed method the actual error (~0.3 dB) of the two-way calibration process was substantially less than the calibration error stated by the manufacturer (1 dB).

1 INTRODUCTION

When calibrating an underwater acoustic transducer for the frequency bandwidth 1 kHz to 500 kHz, without access to equipment such as rotation controllers and an acoustic test tank, usually the transducer needs to be sent to a commercial or standards laboratory for calibration. It is also common to have calibration data supplied when purchasing a transducer. A choice must be made as to whether to conduct a primary (absolute) calibration (IEC 60565 2006, sec. 8) or a secondary (comparison) calibration (IEC 60565 2006, sec. 9) depending on the intended use of the transducer. If the transducer is to be used as a reference transducer to calibrate other transducers the more time-consuming primary method is usually chosen. This is because this method does not require *a priori* knowledge of the sensitivity of another transducer, which is otherwise required in the comparison method. Therefore, the primary method is immune to the propagation of any inherent error in the past calibration of any reference transducer and any uncertainties that arise are only due to stochastic environmental parameters and standard measurement errors at the time of calibration. The primary calibration will then usually have a lower residual uncertainty to that of the comparison calibration method. The primary method can also take a significantly longer duration of time and can be prohibitively expensive when compared to the comparison method, hence the comparison method is most frequently employed to derive transducer sensitivity parameters.

When a third-party is employed to calibrate the transducer, or a transducer is purchased, the standard calibration parameters supplied to the end user are:

1. Complex impedance $Z_T(f)$, and/or its reciprocal, complex admittance $Y(f)$, over the frequency bandwidth Δf.
2. Receive voltage sensitivity $M_{T(dB)}(f)$ and/or the transmit voltage sensitivity $S_{T(dB)}(f)$ over the frequency bandwidth of interest Δf, where usually only either the receive or transmit voltage sensitivity is measured and the reciprocal parameter J is used to obtain its complement (assuming the transducer under test is reciprocal).
3. Normalised beam pattern sensitivities over the direction of interest θ for a particular frequency f for either the receive voltage sensitivity $M_{T(dB)}(\theta)$ or the transmit voltage sensitivity $S_{T(dB)}(\theta)$. The reciprocal parameter J is again used to convert between the receive or transmit voltage sensitivities when only one of these parameters is measured.

These standard calibration parameters are usually derived from the comparison calibration method and the error for the sensitivities is usually quoted as a single number for all frequencies and/or angles, which is often stated as ±1 dB. This broadband error is not reflective of the actual error in calibration at a particular f or θ. This stated precision can be the most limiting factor when attempting to assess precision of measurements undertaken with the calibrated hydrophone. This paper explains that by requesting a two-way comparison calibration, which is inherently a two-way reciprocity test, it is possible to more accurately assess the relative standard uncertainty of the calibration of the hydrophone by just using the supplied parameters $M_{T(dB)}(f)$, $S_{T(dB)}(f)$ and $Z_T(f)$. This technique permits an increase in precision without having to conduct a three-way primary calibration.

2 BACKGROUND THEORY

Although electroacoustic transducer theory has been extensively covered in many texts there is a need for a review of the theory, first to properly explain the concept to assess precision, and in the context of multi element sensing systems. A review follows with updated notation which assists in explaining calibration processes for transducer array systems.

2.1 Electroacoustic Transducer Sensitivity

The electroacoustic transducer sensitivity is a transduction coefficient (Ballantine 1929) that specifies the ability for an electroacoustic transducer to convert between electrical and acoustic quantities. The nominal, or free field, open circuit receive sensitivity M_0 of an underwater transducer is defined by (MacLean 1940):

$$M_0 = \frac{V_0}{P_0} \tag{1}$$

where V_0 is the open circuit output voltage of the transducer, or a similar condition that will produce the same quantity with a negligible outlet current (such as a high impedance measurement device placed across the transducer terminals), P_0 is the effective pressure amplitude in the absence of a measuring transducer at a specified nominal distance which is usually 1m. In underwater acoustics, a particular transducer T has a sensitivity, M_T which is the ratio of M_0 to a reference receive sensitivity M_{ref} and is expressed by the notation $V/\mu Pa$ .The receive sensitivity level $M_{T(dB)}$ of a transducer is usually defined logarithmically as a ratio of M_0 to M_T for a reference value of 1V and is expressed in decibels (Bobber 1970, 181):

$$M_{T(dB)} = 20\log\left(\frac{M_0}{M_{ref}}\right) = 20\log\left(\frac{V_0}{V_{ref}}\right) - 20\log\left(\frac{P_0}{P_{ref}}\right) \tag{2}$$

Since the reference values for voltage and pressure are 1V and 1µPa respectively, $M_{T(dB)}$ is usually calculated as a negative number due to equation (2). The sound pressure level (SPL) at a transducer's terminals is defined as a logarithm of the linear acoustic pressure P :

$$P = \frac{V_{T_{IN}}}{M_T} \qquad \Rightarrow SPL = 20\log\left(\frac{V_{T_O}}{V_{ref}}\right) - M_{T(dB)} \equiv \left|M_{T(dB)}\right| + 20\log\left(\frac{V_{T_O}}{V_{ref}}\right) \tag{3}$$

Where $\left|M_{T(dB)}\right|$ is the conventional notation used in transmission formulae accounting for the negative value for $M_{T(dB)}$ and V_{T_0} is the voltage output at the terminals of the transducer. The receiver amplifier gain G_T of a transducer operating in receive mode needs to be accounted for when making a voltage measurement at the receiver transducer terminals as the amplifier is often integrated into the receiver. Given the transducer receive sensitivity $M^H_{T(dB)}$ the sound pressure level (SPL) at the receiving transducer for an arbitrary sound source at an unknown distance is then:

$$SPL = \left|M^H_{T(dB)}\right| + 20\log\left(V^H_{T_0}\right) - G^H_T \tag{4}$$

where H denotes a transducer operating in receive mode. Similarly, the nominal open circuit transmit sensitivity S_0 is defined by:

$$S_0 = \frac{P_0}{V_0} \tag{5}$$

The reference sensitivity S_{ref} is expressed by the notation of voltage to pressure conversion at a reference distance of 1m; $\mu Pa/V\,@1m$.

The transmit sensitivity of an electroacoustic transducer with transmit sensitivity S_T is similarly also expressed in decibels:

$$S_{T(dB)} = 20\log\left(\frac{S_0}{S_{ref}}\right) = 20\log\left(\frac{P_0}{P_{ref}}\right) - 20\log\left(\frac{V_0}{V_{ref}}\right) \qquad (6)$$

An alternative expression for the free field transmit sensitivity is the current transmit sensitivity S_{T_i} which is the current to pressure conversion at a nominal distance r_0 which is denoted in units; $\mu Pa/A@1m$. Similarly, the SPL produced from a transmitting transducer that is spherically spread at a distance r can be calculated from a measurement of voltage $V_{T_{IN}}$ or current $I_{T_{IN}}$ at the transducer terminals (or if present the amplifier input with a gain of G_T^P), where P denotes a transducer operating in transmit mode and I_{ref} , r_{ref} are 1A and 1m respectively:

$$SPL = S^P_{T(dB)} + 20\log\left(\frac{V^P_{T_{IN}}}{V_{ref}}\right) - 20\log(r) + G_T^P \equiv S^P_{T_i(dB)} + 20\log\left(\frac{I^P_{T_{IN}}}{I_{ref}}\right) - 20\log\left(\frac{r}{r_{ref}}\right) + G_T^P \qquad (7)$$

Given the equivalence in equation (7) the conversion between current and voltage transmit sensitivity is obtained by measurement of the electrical input complex impedance $Z^P_{T_{IN}}$ of the transmitting transducer ($Z_{ref} = 1\Omega$):

$$S^P_{T_i} = \left|Z^P_{T_{IN}}\right| S^P_T \qquad S^P_{T_i(dB)} = S^P_{T(dB)} + 20\log\left(\frac{\left|Z^P_{T_{IN}}\right|}{Z_{ref}}\right) \qquad (8)$$

2.2 Electroacoustic Reciprocity Principle

Electroacoustic reciprocity, which forms the basis of underwater transducer calibration techniques, is derived from Rayleigh's reciprocal theorem (Rayleigh 1945, chap. V) and the application of this theorem whilst observing the reversibility of mutual induction between circuits (Rayleigh 1945, chap. Xb). This was further modified and generalised by Carson (1924) to take into account electrical network theory and radiative fields. Its application to electromechanics and acoustics was developed by Ballantine (1929). Its use in the calibration of spherical electroacoustic transducers was first applied by MacLean (1940) and was further developed by Cook (1941) to derive electromechanical parameters for disk shaped piezoceramic's commonly used in underwater transducers. The principle was theoretically proven in a rigorous examination by Primakoff and Foldy (1947) and its use in underwater transducer calibration is accepted in the calibration standards IEC 60565 (2006) and ANSI/ASA S1.20-2012 (2009).

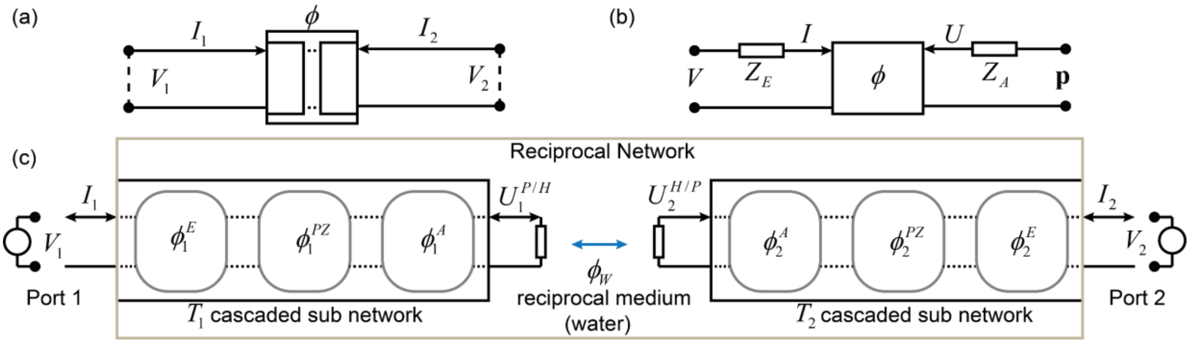

Figure 1:(a) A two port network made of cascaded sub networks representing an electroacoustic transmitter and receiver. (b) Simple two port network representation of electroacoustic transducer with coupling constant ϕ . (c) The greater reciprocal network for a two-way calibration process, which is a cascade of sub networks for both transducers consisting of the electrical matching network ϕ^E , the piezoceramic ϕ^{PZ} , the acoustic matching material covering the transducer ϕ^A and the reciprocal medium for transmission ϕ^W (Arnau Vives 2008, 107).

Electrical reciprocity is illustrated in Figure 1(a) for a simple linear cascaded two port network that is excited by a zero impedance generator V_1 at the input port and I_2 is read by a zero impedance ammeter on the output port. The ammeter and generator switch ports and the reading is repeated for V_2 and I_1 . The network is considered reciprocal if:

$$\frac{I_1}{V_2} = \frac{I_2}{V_1} \qquad (9)$$

The extension of this to an electromechanical circuit (Bobber 1966) is illustrated in Figure 1(b) for a transducer in an arbitrary medium. The receive and transmit sensitivity is then defined by:

$$M_T = \frac{V_0}{\mathbf{p}_m} \quad S_T = \frac{\mathbf{p}_s}{I_{IN}} \tag{10}$$

where $\mathbf{p}_m$ is the average pressure over the transducer area A_m when operating in receive mode, and $\mathbf{p}_s$ is the average far field pressure produced by the transducer operating in transmission mode over some arbitrary area A_S . The transmission (or h parameter) matrix for this two port network is:

$$\begin{bmatrix} \mathbf{p} \\ V \end{bmatrix} = \begin{bmatrix} Z_A & \phi \\ \phi' & Z_E \end{bmatrix} \begin{bmatrix} U \\ I \end{bmatrix} \tag{11}$$

Where $\mathbf{p}$ is either $\mathbf{p}_s$ or $\mathbf{p}_m$ depending on the operation mode of the transducer and U is the net volume velocity emanating from the area A_s or being received over the area A_m . The electroacoustic transfer impedances ϕ and ϕ' are coupling coefficients that account for the dielectric properties of the piezoceramic and acoustic matching layers. For an ideal reciprocal transducer ϕ and ϕ' are equal. The h parameters are obtained by modelling the short-circuit input impedance, open circuit reversed voltage gain, short-circuit forward current gain and open circuit output impedance for Z_A, ϕ, ϕ', Z_E respectively which produces the solutions for transmission and receive modes:

$$\begin{aligned} \mathbf{p}_s &= \phi \mathbf{I}_{IN} \\ \mathbf{V}_0 &= \phi' \mathbf{U} \end{aligned} \tag{12}$$

Multiplying both sides of equation (12) with $\mathbf{p}_m$, eliminating ϕ and then substituting the result into equation (10) produces the result:

$$\frac{M_T}{S_T} = \frac{U_s}{\mathbf{p}_m} \equiv J \tag{13}$$

where J is the *reciprocity parameter* and is the ratio of the receive and transmit sensitivities previously discussed in Section 2.1, and is equivalent to the ratio of the net volume velocity, U_s , emanating from A_s to the resulting pressure $\mathbf{p}_m$ at the transducer.

2.3 Spherical reciprocity parameter

The transducers under test in this paper have spherical elements so only the spherical reciprocity parameter is considered. The reader is directed to Bobber (1966) and Sherman and Butler (2007) for consideration of elements with other geometries. Noting the well-known equation for the pressure from a pulsating sphere at a distance r (Ebaugh and Mueser 1947):

$$\mathbf{p}(r,t) = \frac{j\rho_W c U}{2\lambda r} e^{j(\omega t - kr)} \tag{14}$$

Equation (13) then reduces to the spherical reciprocity parameter J_S for a receiver placed at r :

$$J_S = \frac{2r}{\rho_W f} \tag{15}$$

2.4 Calibration of electroacoustic transducer by the two-way Comparison Method

The calibration of a transducer using the comparison method is achieved by two techniques. The first technique is the substitution method which uses a reference transducer with a known receive sensitivity, M_H^{ref} , placed in the free field of a test source where the voltage response V_O^{ref} is measured. The transducer is then substituted with a test transducer that has an unknown receive sensitivity M_H^{test} and the voltage response V_O^{test} is measured (Bobber 1970, 18). This requires the knowledge of only one transducer's response and results in:

$$M_T^{test} = M_T^{ref} \frac{V_O^{test} d_{test}}{V_O^{ref} d_{ref}} \tag{16}$$

where d_{test} and d_{ref} are included to account for any difference in distances between the source and test and reference transducers respectively. If the transducer is reciprocal in the frequency bandwidth of interest then the parameter J is usually then employed to convert M_T^{test} to the unknown current transmit sensitivity $S_{T_i}^{test}$. Although this method is often quoted as the 'comparison method' in most texts it is logistically difficult to perform as it requires either the exact placement of the substitution transducer or a priori knowledge of the angular distribution of the acoustic energy from the test source. These logistical difficulties can introduce further error into the calibration process.

The second technique, which is the most common comparison method in commercial and standards laboratories, is known as the *projector comparison method*. This method is shown in Figure 2 and involves a more logistically feasible set up where M_T^{test} of a test transducer operating in receive mode can be obtained by knowing the response of a reference transducer in transmit mode S_T^{ref} and observing that the SPL at the test transducer is:

$$\begin{aligned}&\frac{V_{T_O}^{test}}{M_T^{test}} = \frac{V_{T_{IN}}^{ref} S_T^{ref}}{d} \equiv \frac{I_{T_{IN}}^{ref} S_{T_i}^{ref}}{d} \\ &\Rightarrow \frac{1}{M_T^{test}} = \frac{V_{T_{IN}}^{ref} S_T^{ref}}{V_{T_O}^{test} d} \equiv \frac{I_{T_{IN}}^{ref} S_{T_i}^{ref}}{V_{T_O}^{test} d}\end{aligned} \tag{17}$$

Taking the logarithm of both sides of equation (17), including any gain stages of the amplifiers and noting the negative quantity stated for receive sensitivity in equations (3) and (4) results in:

$$\left|M_{T(dB)}^{test}\right| = S_{T(dB)}^{ref} + 20\log\left(\frac{V_{T_{IN}}^{ref}}{V_{T_O}^{test}}\right) - 20\log\left(d\right) + G_T^{test_H} + G_T^{ref_P} \tag{18}$$

where $G_T^{test_H}$ and $G_T^{ref_P}$ are the gain of the amplifiers for the test and reference transducers respectively. The transmit voltage sensitivity of the test transducer $S_{T(dB)}^{test}$ can be inferred by using the reciprocal parameter J in equation (15) and the impedance of the test transducer in equation (8). Alternatively, a physical measurement of $S_{T(dB)}^{test}$ can be achieved by keeping both transducers in place and now employing the reference transducer in receive mode, and the test transducer in transmit mode, which is a reversal of notation for reference and test transducers in equation (18). This reversal results in the equation:

$$S_{T(dB)}^{test} = \left|M_{T(dB)}^{ref}\right| + 20\log\left(\frac{V_{T_O}^{ref}}{V_{T_{IN}}^{test}}\right) + 20\log\left(d\right) - G_T^{test_P} - G_T^{ref_H} \tag{19}$$

Both equation (18) and (19) define the *two-way comparison method* for physical measurement of the receive and transmit sensitivity of an underwater electroacoustic transducer.

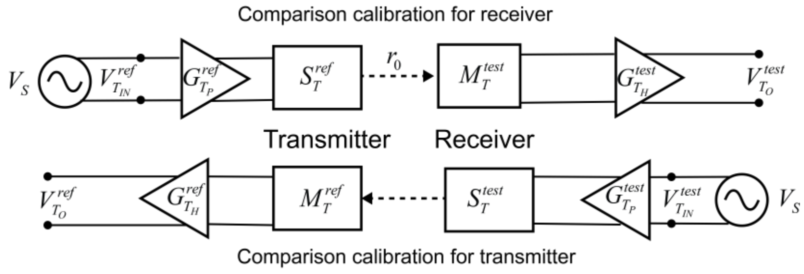

Figure 2: Schematic for a two-way comparison calibration process

2.5 Electroacoustic transducer reciprocity test

Two transducers may be tested for reciprocity whilst conducting a two-way comparison calibration test. Observing Figure 1(a) and equation (9), two linear transducers in a two-way calibration process within a reciprocal medium resembles a greater reciprocal network described in Figure 1(c). The two transducers, which occupy opposite ports of the network are reciprocal if in a two-way transmission process:

$$\frac{V_{T_O}^{H1}}{I_{T_{IN}}^{P2}} = \frac{V_{T_O}^{H2}}{I_{T_{IN}}^{P1}} \tag{20}$$

where $V_{T_O}^{H1}$ and $I_{T_{IN}}^{P1}$ are the voltage output and current input for transducer 1 acting in receive and transmit mode respectively. Similar notation is applied for transducer 2.

2.6 Propagation of errors

The evaluation of the combined standard uncertainty, $u_c\left(y\right)$, of a multivariate function $y = f\left(x_i\right)$ is achieved by application of the *general law of error propagation* (Ku 1966) which is a first-order Taylor series approximation of small deviations of u about μ_u for the measurand y. This results in a linear sum of terms of partial differentials which represents the variation of the output estimate y with the standard uncertainty of each input estimate x_i (JCGM 2008, 21):

$$
\begin{aligned}
u_c^2(y) &= \sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}u(x_i)u(x_j)r(x_i,x_j) \\
&= \sum_{i=1}^{\mathrm{N}} c_i^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} c_i^2 c_j^2 u(x_i)u(x_j)r(x_i,x_j)
\end{aligned}
\tag{21}
$$

Where $u(x_i)$ are the individual standard uncertainties (Type A or B, (Hall 2016, 51)) of each input x_i, c_i the sensitivity coefficients and $r(x_i,x_j)$ is the correlation coefficient between each input x_i and is defined by:

$$
r(x_i,x_j) = \frac{u(x_i,x_j)}{u(x_i)u(x_j)} \qquad -1 \le r(x_i,x_j) \le 1 \tag{22}
$$

If x_i and x_j are independent then the correlation coefficient reduces to zero and the covariance term in equation (21) vanishes. In such a case for the multivariate function:

$$
y = f(x_i) = cx_1^{P_1}cx_2^{P_2}cx_3^{P_3}\ldots cx_N^{P_N} \tag{23}
$$

where the exponents p_i are real numbers, it can be shown using equation (21) that the square of the relative combined standard uncertainty is the sum of square of all relative standard uncertainties of the measured inputs: (JCGM 2008, 20):

$$
\left(\frac{u_C(y)}{|y|}\right)^2 = \sum_{i=1}^{N}\left[\frac{p_i u(x_i)}{|x_i|}\right]^2 \tag{24}
$$

3 COMPARISON CALIBRATION OF RESON TC 4034

Calibration of a Reson TC4034 occurred at the facilities of Neptune Sonar Plc. in the East Yorkshire district of the UK over the period of the 28th and 29th of July, 2016. The calibration facilities comprise of a floating pontoon in a quarry lake (Lake Kelk) which houses a laboratory monitoring several reference hydrophones spread over various positions on the pontoon platform (Figure 3). The period for calibration is considered the warmest part of the year in the UK and was chosen to be comparable to the expected temperatures for the operation of the transducer in Perth, Australia. As there are similar temperatures for the calibration and operational environments for the transducers they will not require any temperature offset adjustment (Van Buren, Drake, and Paolero 1999). The temperature of the water was stable at 20º C during the entire duration of testing. The depth of the lake is approximately 9m with the hydrophones set at mid depth at 4.5m to establish free field conditions. The two-way comparison method described in section 2.4 was employed where the reference hydrophones used for the comparison were pre-calibrated using the three-way primary calibration method described by (Ebaugh and Mueser 1947; IEC 60565 2006, 41). The arrangement of the hydrophones on the pontoon is shown in Figure 3 which also illustrates the distances for transmission and the bandwidths employed for each reference hydrophone.

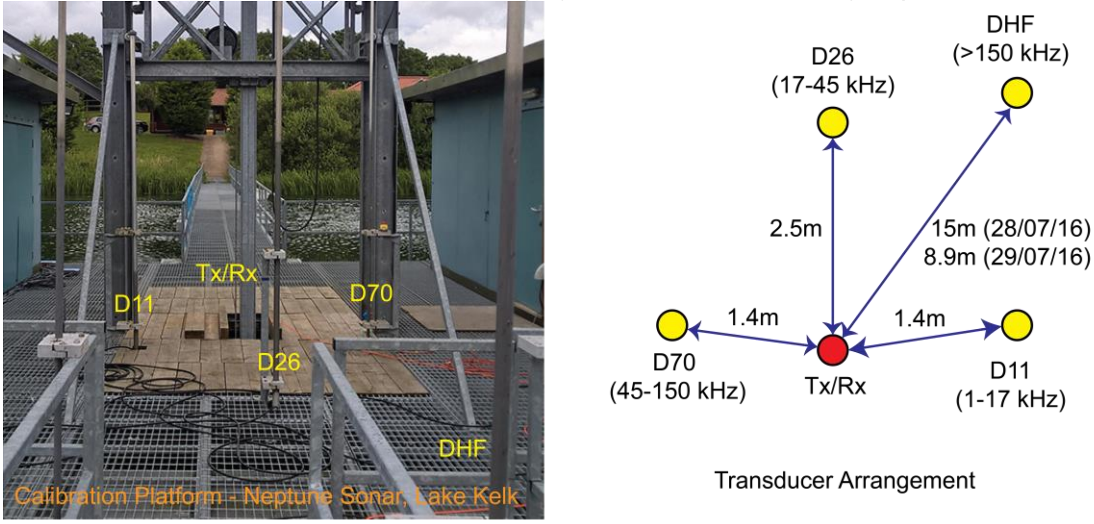

Figure 3: Neptune Sonar Calibration Facilities, Lake Kelk, UK and hydrophone placement and frequency bandwidths employed for each reference transducer

3.1 Comparison Calibration Procedure

The calibration procedure first employed a HP 4192a Low Frequency Impedance Analyzer to measure the complex electrical impedance and admittance at 1 kHz intervals whilst water loaded. The results are shown in Figure 4 with emphasis on the expected operational bandwidth of the transducer (50-90 kHz).

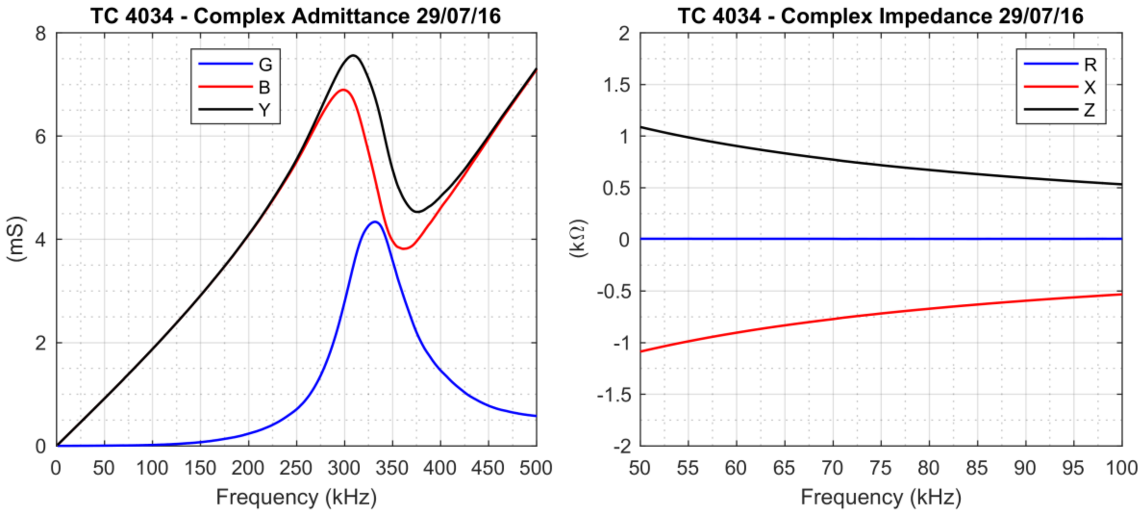

Figure 4: Complex admittance and impedance curves for the TC4034

Sensitivity curves were then established for the TC4034 at the 0° reference point on the transducer acting as a transmitter and then remeasured whilst as a receiver using an automated calibration signal path (Figure 5). The TC4034 was driven as the source under test using a B&K 2713 current amplifier connected to a HP 33120a Signal Generator. The signals employed were narrow-band sinusoids consisting of 5 pulses of 10-30 cycles (duration specific) at a chosen frequency. This duration was enough to achieve steady state conditions for amplitude measurement. The reference receiver was externally gated by the HP 33120a and was measured by a HP 9410a Vector Analyser which also referenced the output of the B&K 2713 via a 40dB attenuated tap line. Signal processing occurred on a PC connected to the HP 8941a and HP 33120a via a GPIB link.

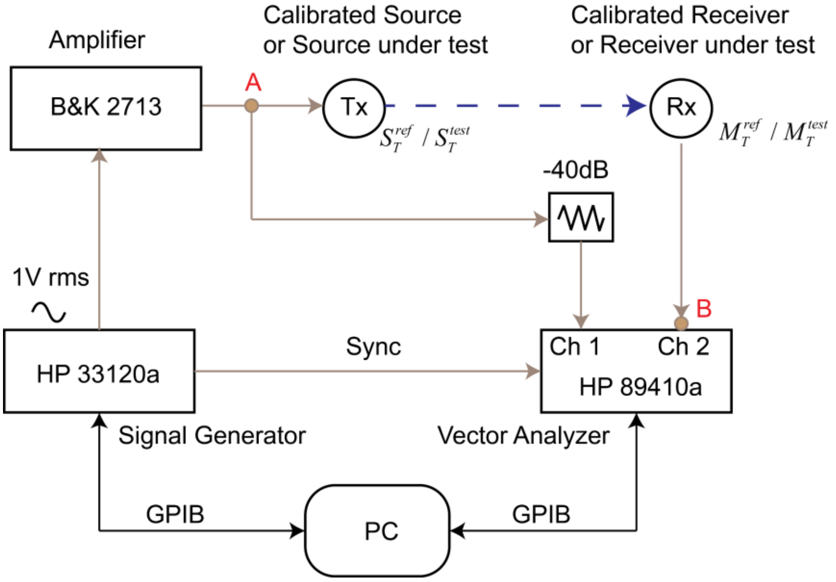

Figure 5: Signal flow path for two-way comparison calibration at Neptune Sonar Plc.

The sensitivity was measured every 1 kHz within the interval 10-150 kHz and then every 5 kHz within the interval 150-400 kHz. When testing the TC4034 as a receiver the transducers were kept in place and cables were swapped between the channel 2 input of the HP 89410a to the output of the B&K 2713 (nodes A and B, Figure 5) and the test sequence was repeated. The results are shown in Figure 6 with emphasis on the expected operational bandwidth. The magnitude response is considered linear in the expected operational range of the transducer.

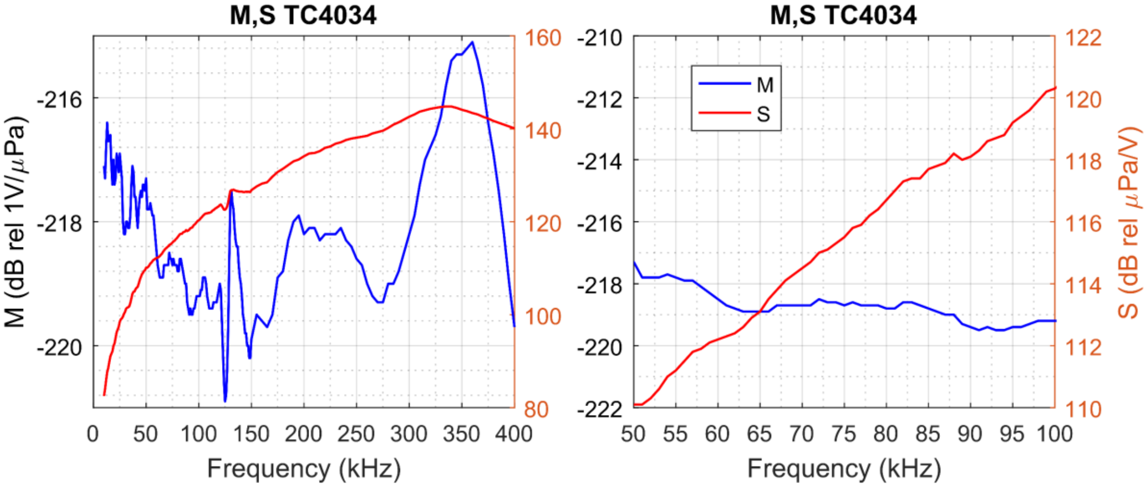

Figure 6: Transmit and receive sensitivity curves for the Reson TC4034 (figures use common legend)

3.2 Determination of the calibration error

The calibration error supplied by Neptune Sonar states the sensitivity uncertainty is ±1 dB for each frequency measurement. The calibration certificates were also obtained for the reference transducers used in the calibration of the TC4034 and no errors were quoted. The errors arising during the course of the calibration are mainly due to the existing errors for the values M_T^{ref} and S_T^{ref} for the reference transducers in equation (18) and (19), denoted ΔM_T^{ref} and ΔS_T^{ref}. Since the distance between transducers on the pontoon and voltage and gain values can be determined with a high degree of precision they contribute less to the combined uncertainty. The calibration process is automated so it is not possible to inspect the voltage values produced at the reference transducers to observe any variance in these values dues to stochastic fluctuations in electrical noise or the environment. It is possible however, to obtain an upper limit on $\Delta M_{T(dB)}^{test}$ and $\Delta S_{T(dB)}^{test}$ when conducting the two-way comparison calibration by noting that the process described is inherently a two-way reciprocity test depicted in Figure 1(c). Rearranging the reciprocal parameter in equations (13) and (15) and converting the current to voltage transmit sensitivities in equation (8) yields unity (noting the conversion for $M_{T(dB)}^{test}$ and $S_{T(dB)}^{test}$ from µPa to Pa):

$$C(f)=\frac{M_T^{test}(f)}{S_T^{test}(f)\left|Z_{T_{IN}}^{test}(f)\right|J_T^{test}(f)}\left(1\times10^{12}\right)=\frac{\rho_W f}{2r_0\left|Z_{T_{IN}}^{test}(f)\right|}10^{\frac{\left(M_{T(dB)}^{test}(f)-S_{T(dB)}^{test}(f)\right)}{20}+12}\equiv 1 \tag{25}$$

This unity relationship is denoted here as the Reciprocity Constant C for the standard reference distance of 1m. Equation (25) can then be plotted over frequency (Figure 6) to investigate whether the TC4034 is a reciprocal transducer over the tested bandwidth. In deriving the standard uncertainty for the reciprocity constant $\Delta C(f)$, and noting that M_T^{test} and S_T^{test} have been measured independently of each other, application of equation (24) with equation (25) yields the sum of squares of all relative standard uncertainties of measured inputs:

$$\left(\frac{\Delta C(f)}{|C(f)|}\right)^2=\left(\frac{\Delta M_T^{test}(f)}{\left|M_T^{test}(f)\right|}\right)^2+\left(\frac{\Delta S_T^{test}(f)}{\left|S_T^{test}(f)\right|}\right)^2+\varepsilon^2 \tag{26}$$

Where ε represents the relative standard uncertainties for measurement of frequency, density of water, and test transducer impedances. Figure 6 then illustrates the inclusion of the error range for $C(f)$ in yellow using the stated ±1 dB error (~0.12%) from Neptune Sonar. If this ±1 dB error was true the value for $C(f)$ should range over the yellow shaded area which it doesn't appears to do so. Further inspection of equation (26), assuming a very small value for ε, reveals that the standard uncertainties (when rounded) form a Pythagorean triplet such that:

$$\frac{\Delta C(f)}{|C(f)|}>\left\{\frac{\Delta M_T^{test}(f)}{\left|M_T^{test}(f)\right|},\frac{\Delta S_T^{test}(f)}{\left|S_T^{test}(f)\right|}\right\}\in f_{measured} \tag{27}$$

This result implies that any of the relative standard uncertainties of M_T^{test} and S_T^{test} will be less than the relative standard uncertainty of $C(f)$ over the measured bandwidth $f_{measured}$. Noting this, the relative standard uncertainty for $C(f)$ can then be converted to dB and is shown in Figure 6. The conversion indicates that the maximum value for $\Delta M_{T(dB)}^{test}$ or $\Delta S_{T(dB)}^{test}$ which is approximately ±0.3 dB and is far less than the stated uncertainty of ±1 dB. If ε is of significance then the observation is still valid as the relative standard uncertainties summed in quadrature will always be individually less than the combined relative uncertainty.

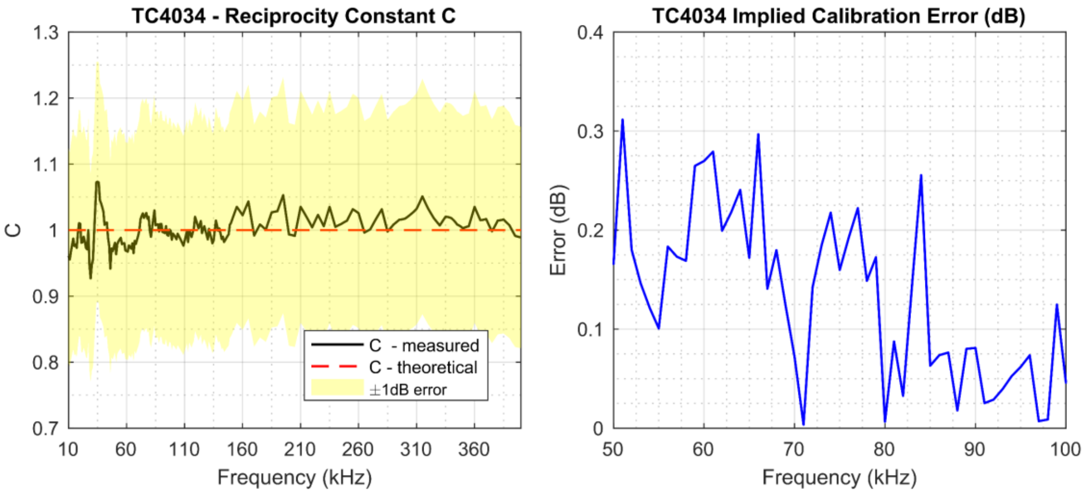

Figure 7: The Reciprocity Constant for the measured bandwidth and its relative error in dB. The yellow shaded area depicts the expected range for C using the manufacturer's error.

4 DISCUSSION

The standard uncertainties $\Delta M_{T(dB)}^{test}$ and $\Delta S_{T(dB)}^{test}$ assume that they are independent measurable variables. It is important to note that from equation (17) that M_T^{test} and S_T^{test} are composite functions of M_T^{ref} which has been obtained by three-way primary reciprocity calibration and its equation is of the form (Bobber 1970, 29):

$$M_T^{ref} = \sqrt{\left(\frac{e_{TH}e_{PH}}{e_{PT}i_T}\frac{d_1'}{d_0'}J_T^{ref'}\right)} \tag{28}$$

where e_{XX} are the voltages seen across the terminals in the three-way process for projector, d' is the distances used in the three-way process, P, T and H are the transducers used for projection, reciprocal transmission and receiver under test, i_T the input current to the reciprocal transducer and $J_T^{ref'}$ the reciprocal parameter used for the reciprocal transducer. There is a possibility that the reference transducer used in the comparison calibration process has had reciprocal parameter J_T^{ref} employed to convert M_T^{ref} to $S_{T_i}^{ref}$ and S_T^{ref}. Although it is not possible to ascertain if this was performed for the reference transducer discussed in this paper, if it is known that this has occurred then M_T^{test} and, $S_{T_i}^{test}$ using equation (17), become:

$$M_T^{test} = \frac{V_{T_O}^{ref} Z_T^{ref} J_T^{ref} d}{V_{T_{IN}}^{ref} M_T^{ref}} \qquad S_{T_i}^{test} = \frac{V_{T_O}^{ref} Z_T^{ref} d}{V_{T_{IN}}^{ref} M_T^{ref}} \tag{29}$$

Given that the ratio of these two measured variables equals the reciprocal parameter J_T^{test} it is easy to arrive at the erroneous assumption that all measured inputs cancel out making assessment of the contributing variables to the combined uncertainty for $C(f)$ difficult. Although the measured inputs cancel out, the uncertainties of the measured inputs do not as they are measured in each stage of the two-way process. The two-way process can be considered a separate snapshot of Type A and Type B uncertainties for each measurement in the two-way process. When considering the variables contributing to the combined uncertainty, voltage, current and impedance measurements are subject to Type A stochastic errors and Type B measurement offset errors. Additionally as shown in equation (13), J_T^{test}, J_T^{ref} and $J_T^{ref'}$ are ideal representations of the ratio of the net volume velocity emanating from a reference transducer in its far field to the pressure arriving at the transducer under test. This representation is not correct as nonlinearities exist in the coupling coefficients illustrated in Figure 1(c) which further contribute to the combined uncertainty. This reasoning illustrates that even if the reference transducer was treated as reciprocal then the relative standard uncertainty for $C(f)$ is still a measurable quantity with independent variables with likely little or no covariance between them. The deduction of an implied error for M_T^{test} and S_T^{test} is still valid. It must also be noted that the maximum implied error of ±0.3 dB (~1%) may be higher or lower as the two-way transmission calibration has only been performed once. Although the results are indicative of the implied error being less, further tests would need to be conducted to confirm this. Additionally, the uncertainty derived for this calibration seems to be about half the uncertainty measured in a recently documented three-way reciprocal calibration process for another TC4034 (Crocker 2016). Further investigation is warranted to account for this discrepancy.

5 CONCLUSION

A two-way comparison calibration process was conducted and the reciprocal nature of the two transducers involved were examined to derive an *implied* calibration error of ±0.3dB for a Reson TC4034, which is less than the stated calibration error of ±1dB in the supplied calibration certificate. The technique presented may increase the precision of comparison calibration operations for underwater electroacoustic transducers without having to resort to a more time-consuming and costly primary three-way reciprocity technique.

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