A REVISION OF A PREVIOUS DISCUSSION ON THE CORRECTNESS OF CURLE'S AND THE FFOWCS WILLIAMS AND HAWKINGS EQUATIONS

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Abstract

It is well known that noise generated by fluid flow around propellers and hulls of maritime platforms significantly contributes to their acoustic signature. The level of this noise is often determined by means of Curle's and the Ffowcs Williams and Hawkings (FW-H) equations, according to which the noise level depends on the total force acting upon the rigid boundary and the velocity of the boundary. The author and his previous co-author claimed that these equations are incorrect and suggested another equation where the noise level was determined by the acoustic pressure and its normal derivative, i.e. the potential component of velocity, on the boundary. The purpose of this paper is to reconsider the arguments which the author and his co-author presented during the discussion on the correctness of these equations. It is shown that the FW-H equation in the integral form and the equation derived previously by the present author can be obtained from the FW-H equation in the differential form utilising different ways of evaluating integrals of the source terms. It is demonstrated that the equation derived by the author does not contradict the FW-H and Curle's equations and is another form of these equations expressed via a different set of variables. It is concluded that viscous stresses as well as the rotational component of velocity do not contribute to the acoustic radiation from a rigid boundary in fluid flow.

1. Introduction

Reliable methods for predicting the level of noise radiated by fluid flows are important in many areas of science and engineering. For instance, they are invaluable for minimising acoustic signatures of maritime platform and weapons such as ships, submarines and torpedos. They are also used in the development of helicopter rotors, aircraft fuselages and landing gear as well as bodies of automobiles. The first breakthrough in understanding the mechanism of acoustic noise generated by turbulent flow was achieved by Lighthill [1]. He showed that, if there are no boundaries, the amplitude of such noise is determined by Lighthill's stress tensor. He also showed that the radiated sound had quadrupole characteristics.

Curle extended Lighthill's theory to the case of turbulent flow near a solid stationary boundary [2]. He showed that the radiated sound in this case consists of Lighthill's quadrupole sound as well as dipole sound, which amplitude was determined by the total force acting upon the fluid from the boundary.

Later, Ffowcs Williams and Hawkings derived another equation for the amplitude of sound radiated by fluid flow near a boundary [3]. This equation is an exact rearrangement of mass and momentum conservation equations and, therefore, can be applied to general flow near stationary or moving boundaries. For a stationary boundary, the Ffowcs Williams and Hawkings (FW-H) equation reduces to Curle's equation. In addition to the Lighthill's quadrupole acoustic sources in the fluid volume and Curle's dipole sources on the boundary, the FW-H equation also describes monopole sources determined by the normal velocity of the boundary. Since its derivation, the FW-H equation has become one of the most frequently used methods for predicting characteristics of acoustic flow noise.

Zinoviev and Bies [4] reconsidered Curle's derivation and claimed that some mathematical transformations in this derivation should have been done differently. They obtained an equation which differed from Curle's and the FW-H equations by the appearance of its source terms that were determined only by acoustic components of stresses and normal velocity on the boundary. They also provided some examples which, as the authors believed at the time, proved that the FW-H equation did not describe the sound generation correctly. These claims caused objections from some members of the aeroacoustical community [5,6]. The authors of these objections stated that the examples provided in Ref [4], in fact, confirmed the correctness of the FW-H equation. Farassat [5] stated that the equations derived by Zinoviev and Bies in Ref [4] is equivalent to the linearized FW-H equation when the rigid boundary is in motion with small amplitude. Zinoviev and Bies provided their responses to this criticism [7,8], where they insisted that their entire initial claims were valid. Recently the authors have retracted their statements that Curle's and the FW-H equations are incorrect [9], but still affirmed that the equation derived by them in Ref. [4] is valid and fully equivalent to the FW-H equation, not only to its linearized form.

In two conference reports [10,11], Zinoviev considered a question whether Curle's and the FW-H equations in their integral forms satisfy the same wave equation and boundary conditions as the equation derived in Ref. [4], i.e. whether the latter equation is just another form of the former ones. Note that in Refs. [10, 11] the equation derived in Ref. [4] is called "the non-uniform Kirchhoff equation". In this paper, this equation is called "inhomogeneous Kirchhoff equation", as this name better describes the appearance of the equation, which source terms include Kirchhoff surface integrals [12] together with the term that determines Lighthill's quadrupole sound.

In Ref. [10], it was shown that the difference in the source terms of Curle's and the inhomogeneous Kirchhoff equations is limited to a sum of two surface integrals with the integrands depending on Lighthill's stress tensor. Based on this fact, it was suggested that, if the sum of the integrals was exactly zero, the two equations were different forms of one equation. In Ref. [11], it was shown that the inhomogeneous Kirchhoff equation can be obtained using the derivation employed by Ffowcs Williams and Hawkings if another set of equivalent boundary conditions is used.

As the question of the validity of the inhomogeneous Kirchhoff equation remains without a proper answer, the purpose of this paper is to present an argument about the similarity between this equation, the FW-H equation, and Curle's equation. In Section 2, the derivation of the FW-H equation in the differential form carried out by Ffowcs Williams and Hawkings [3] is reviewed. In Section 3, Kirchhoff formula for a solution of an inhomogeneous wave equation is presented. Section 4 demonstrates how the FW-H equation in the differential form can be solved with the use of Kirchhoff formula resulting in the FW-H equation in the integral form. Section 5 is devoted to the derivation of the inhomogeneous Kirchhoff equation, and the relationship between the FW-H equation in the integral form and the inhomogeneous Kirchhoff equation is investigated in Section 6.

2. Review of the derivation of the FW-H equation in the differential form

In their derivation, Ffowcs Williams and Hawkings [3] considered a fluid volume, V, divided into regions 1 and 2 by a surface, S, moving in the region 2 with the velocity, $\mathbf{v}$ (Figure 1). Volumes V_1 and V_2 refer to the corresponding regions. The surface S was assumed to be a surface where the vectors of mass and momentum flows could become discontinuous.

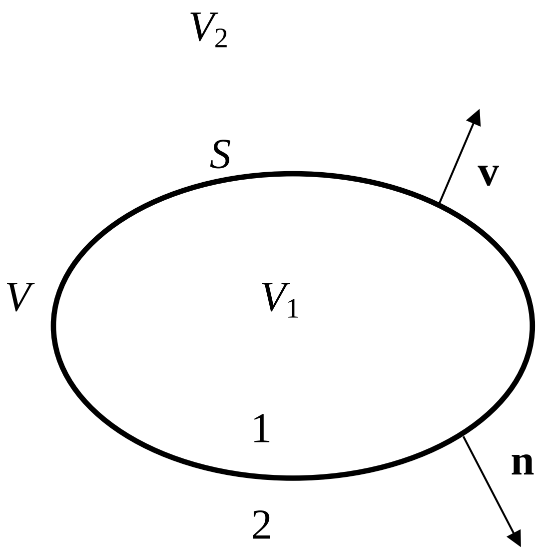

Figure 1. Layout of the fluid volume, *V*, with the moving surface of discontinuity, *S*

In this analysis, Einstein summation over repeating indices is assumed. According to this notational convention, repeated indices in every term are implicitly summed over. Utilising indices $i, j = 1, 2, 3$, which correspond to the three spatial coordinates (x_1, x_2, x_3), the continuity and momentum equations obtained by Ffowcs Williams and Hawkings can be written, respectively, as follows:

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x_i}(\bar{\rho}\bar{u}_i) = \left(\rho(u_i - v_i)\right)^{(2)} n_i \delta(S) - \left(\rho(u_i - v_i)\right)^{(1)} n_i \delta(S), \tag{1}$$

$$\frac{\partial}{\partial t}(\bar{\rho}\bar{u}_i) + \frac{\partial}{\partial x_j}\left(\bar{\rho}\bar{u}_i\bar{u}_j + \bar{p}_{ij}\right) = \left(p_{ij} + \rho u_i\left(u_j - v_j\right)\right)^{(2)} n_j \delta(S) - \left(p_{ij} + \rho u_i\left(u_j - v_j\right)\right)^{(1)} n_j \delta(S), \tag{2}$$

where p_{ij} is the compressive stress tensor defined as

$$p_{ij} = p\delta_{ij} + \mu\left(-\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} + \frac{2}{3}\left(\frac{\partial u_k}{\partial x_k}\right)\delta_{ij}\right). \tag{3}$$

In Eqs. (1) to (3), ρ is the fluid density, **u** is the fluid particle velocity, *t* is time, **n** is the unity vector normal to *S* directed towards the fluid, *p* is the pressure, μ is the dynamic viscosity of the fluid, and δ_{ij} is the Kronecker's delta. The indices 1 and 2 refer to values in the corresponding regions, overbar above a variable implies that the value for this variable can be taken in any region, and $\delta(S)$ is the three-dimensional Dirac delta function determined by the following equation:

$$\delta(S) = \delta(x_1 - \tilde{x}_1)\delta(x_2 - \tilde{x}_2)\delta(x_3 - \tilde{x}_3), \quad (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) \in S, \tag{4}$$

where $\delta(x)$ is the one-dimensional Dirac delta-function [13].

It is clear that Eq. (1) represents the mass conservation law for a small fluid volume. Indeed, the first term on the left is the change of mass in the volume, the second term on the left is the mass flow divergence, i.e. the mass flow from the small volume towards the outside fluid, and the right-hand part represents the mass flow from the surface S to the small volume. Due to the presence of the delta-function $\delta(S)$, the latter mass flow exists only if the small volume intersects with the surface S.

Likewise, Eq. (2) is the momentum conservation law for the same small fluid volume. In fact, it represents three equations for three components for the momentum vector $\rho\mathbf{u}$, each of which corresponds to one value of the index i=1,2,3. In Eq. (2), the first term on the right is the change of momentum of the small fluid volume, the second term on the right taken with the opposite sign is the force acting on this volume from the fluid, and the right-hand part is the force acting on the volume from the surface S. It may be noted that the total force consists of three terms. The term $p\delta_{ij}$ is the acoustic pressure; the term proportional to the viscosity of the fluid μ represents the viscous stresses; and the term $\bar{\rho}\bar{u}_i\bar{u}_j$ determines Reynolds stresses due to turbulent fluctuations of the momentum of the fluid.

In this analysis, it is assumed that the fluid particle velocity u and the velocity of the boundary v are much smaller than the speed of sound, c_0, in the fluid, so that Mach number, M, is considered to be equal to unity. It is necessary to note that this assumption is made in order to avoid unnecessary complexity and does not mean that the analysis is reduced to the linear case, as all nonlinear terms with respect to the stress, density and velocity fluctuations are taken into account.

To derive an equation for the sound amplitude from Eqs. (1) and (2), it is necessary to define boundary conditions on both sides of the surface S. Although the surface of integration in the FW-H method can also be permeable [14], the present analysis deals with the most common case where the surface is considered to be impenetrable. Therefore, it is logical to consider the first boundary condition to be the equality of the normal component of the velocity of the fluid in the outside region and that of the surface:

$$u_n^{(2)}\Big|_S = v_n. \tag{5}$$

In addition, Ffowcs Williams and Hawkings made an assumption that the fluid inside the rigid object is at rest and, as a result, all thermodynamic variables in that region had their equilibrium values. This led to the following boundary conditions for the region 1:

$$\rho^{(1)} = \rho_0 = \text{const}, \tag{6}$$

$$p_{ij}^{(1)} = 0. \tag{7}$$

In Eq. (7), p_{ij} is interpreted as the difference of the stress tensor from its mean value.

By substituting Eqs. (5)–(7) into Eqs. (1) and (2) and by performing some mathematical manipulations Ffowcs Williams and Hawkings obtained the following inhomogeneous wave equation for the density fluctuations $\rho' = \rho - \rho_0$ in the fluid:

$$\left(\frac{\partial^2}{\partial t^2} - c_0^2\frac{\partial^2}{\partial x_i^2}\right)\bar{\rho}' = \frac{\partial^2 \bar{T}_{ij}}{\partial x_i \partial x_j} - \frac{\partial}{\partial x_i}\left(p_{ij}\delta(S)n_j\right) + \frac{\partial}{\partial t}\left(\rho_0 v_i \delta(S) n_i\right), \tag{8}$$

where T_{ij} is Lighthill's stress tensor, which is determined as

$$T_{ij} = \rho u_i u_j + p_{ij} - c_0^2\rho'\delta_{ij}. \tag{9}$$

In Eq. (9), the first and second terms in the right-hand part represent the total stress in the fluid, whereas the third term with the positive sign is the stress due to compressibility of the fluid, i.e. the acoustic pressure. Therefore, it can be said that Lighthill's stress tensor denotes all non-acoustic stresses that include the viscous and Reynolds stresses.

Equation (8) is valid for the total volume V that includes both regions 1 and 2 as well as the surface S. All acoustic variables have the expected values outside the surface and zero values inside the surface. Eq. (8) is the FW-H equation in the differential form.

3. Kirchhoff formula for the solution of a wave equation

The FW-H equation in the integral form is a solution of Eq. (8), which is an inhomogeneous wave equation. This solution can be obtained with the use of Kirchhoff formula (Section 9.7.6 of the Reference [15]). Assume that a closed surface S separates the total infinite volume V into the internal and external volumes, V_1 and V_2 (Figure 2).

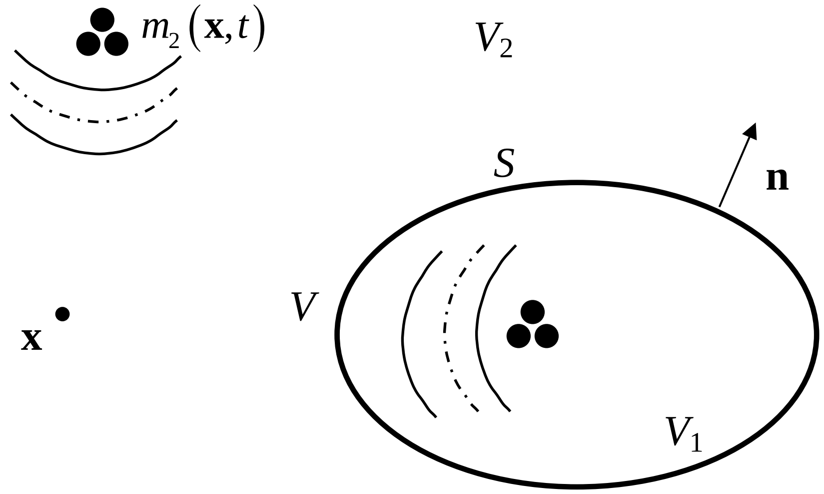

Figure 2. Application of Kirchhoff formula to a wave equation. V_1 and V_2 are respectively the internal and external volumes to a surface S; $m_1(\mathbf{x},t)$ and $m_2(\mathbf{x},t)$ are sources of an unknown potential field $\phi(\mathbf{x},t)$ that are located in V_1 and V_2; $\mathbf{x}$ is the observation point; and $\mathbf{n}$ is the normal unity vector external to S.

Assume also that $m_1(\mathbf{x},t)$ and $m_2(\mathbf{x},t)$ are sources of an unknown potential field that are located inside the volumes V_1 and V_2 respectively. The potential field is described by a function $\phi(\mathbf{x},t)$. Then, according to Kirchhoff formula, in the absence of sources at infinite distance from S any solution of the following inhomogeneous wave equation for the function $\phi(\mathbf{x},t)$,

$$\left(\frac{\partial^2}{\partial t^2}-c_0^2\frac{\partial^2}{\partial x_i^2}\right)\phi(\mathbf{x},t)=m_2(\mathbf{x},t), \tag{10}$$

in the external volume, V_2, can be determined as follows:

$$4\pi\phi(\mathbf{x},t)=\frac{1}{c_0^2}\iiint_{V_2}\frac{[m_2]}{r}dV(\mathbf{y})-\iint_S\left\{\frac{1}{r}\left[\frac{\partial\phi}{\partial n}\right]-[\phi]\frac{\partial}{\partial n}\left(\frac{1}{r}\right)+\frac{1}{rc_0}\left[\frac{\partial\phi}{\partial t}\right]\frac{\partial r}{\partial n}\right\}dS(\mathbf{y}). \tag{11}$$

In Eq. (11), $r=|\mathbf{x}-\mathbf{y}|$ is the distance between the observation point, $\mathbf{x}=(x_1,x_2,x_3)$, and the source point, $\mathbf{y}=(y_1,y_2,y_3)$, $\partial/\partial n=n_i\,\partial/\partial y_i$ is the derivative over the direction of the unity normal vector $\mathbf{n}$, and the square brackets denote the dependence on the retarded time, $\tau=t-r/c_0$. Due to this dependence, it can be proven that the following equation is correct:

$$\frac{\partial}{\partial y_i}\left\{\frac{1}{r}[\phi]\right\} = -\left\{\frac{1}{r^2}[\phi] + \frac{1}{c_0 r}\frac{\partial[\phi]}{\partial t}\right\}\frac{\partial r}{\partial y_i}. \tag{12}$$

As a result, Eq. (11) can be re-written as follows:

$$4\pi\phi(\mathbf{x},t) = \frac{1}{c_0^2}\iiint\limits_{V_2}\frac{[m_2]}{r}dV + \iint\limits_{S} n_i\frac{\partial}{\partial y_i}\left(\frac{[\phi]}{r}\right)dS(\mathbf{y}) - \iint\limits_{S} n_i\frac{1}{r}\frac{\partial[\phi]}{\partial y_i}dS(\mathbf{y}). \tag{13}$$

Equation (13) is the solution of the inhomogeneous wave equation (Eq. (10)). The first term in the right-hand side of Eq. (13) determines the field generated by the volume distribution of sources with the source density $m_2(\mathbf{x},t)$ *inside* the volume V_2. The second and third terms on the right in Eq. (13) determine the field generated by sources *outside* the volume V_2, i.e. in the volume V_1. In acoustics, the function $\phi(\mathbf{x},t)$ represents distribution of either the velocity potential or the density fluctuations in an acoustic wave that is generated by the acoustic sources determined by the function $m_2(\mathbf{x},t)$.

4. Derivation of the FW-H equation in the integral form

In the case under consideration there are no acoustic sources at infinite distance from the surface S, so that the Kirchhoff formula (Eq. (13)) can be applied to obtain the solution of Eq. (8) in the external volume V_2. It can be seen that the right-hand part of Eq. (8) describes the acoustic sources in the whole volume V, which includes the interior volume V_1, the exterior volume V_2 as well as the surface S separating the two volumes. Indeed, the first term on the right in Eq. (8) determines the acoustic sources due to Lighthill's stress tensor T_{ij} in the exterior volume V_2, the second and third terms on the right determine the sources on the surface S due to the field discontinuities on S, and the sources in the interior volume V_1 are absent due to the assumption about the equilibrium field state in V_1 (Eqs. (6) and (7)). Therefore, when applying the Kirchhoff formula to Eq. (8), it is sufficient to consider only the volume integral (the first term in Eq. (13)), which should be taken over the total volume V. The resulting solution of Eq. (8) takes the following form:

$$4\pi c_0^2\rho'(\mathbf{x},t) = \iiint\limits_{V}\frac{1}{r}\left\{\frac{\partial^2[T_{ij}]}{\partial y_i \partial y_j} - \frac{\partial}{\partial y_i}\left([p_{ij}n_j]\delta(S)\right) + \frac{\partial}{\partial t}\left(\rho_0[v_i n_i]\delta(S)\right)\right\}dV(\mathbf{y}). \tag{14}$$

As the second and third terms on the right-hand side of Eq. (8) contain Dirac delta functions, the volume integrals of these terms over V are reduced to surface integrals over S and Eq. (14) takes the form of

$$4\pi c_0^2\rho'(\mathbf{x},t) = \iiint\limits_{V_2}\frac{1}{r}\frac{\partial^2[T_{ij}]}{\partial y_i \partial y_j}dV(\mathbf{y}) - \iint\limits_{S}\frac{1}{r}\frac{\partial[p_{ij}n_j]}{\partial y_i}dS(\mathbf{y}) + \rho_0\frac{\partial}{\partial t}\iint\limits_{S}\frac{[v_n]}{r}dS(\mathbf{y}). \tag{15}$$

The first and second terms in the right-hand side of Eq. (14) are convolutions, so that the derivatives can be interchanged with the integrals [16, p. 126]. As a result, the following equation is obtained as a solution of Eq. (8):

$$4\pi c_0^2\rho'(\mathbf{x},t) = \frac{\partial^2}{\partial x_i \partial x_j}\iiint\limits_{V_2}\frac{[T_{ij}]}{r}dV(\mathbf{y}) - \frac{\partial}{\partial x_i}\iint\limits_{S}\frac{[p_{ij}n_j]}{r}dS(\mathbf{y}) + \rho_0\frac{\partial}{\partial t}\iint\limits_{S}\frac{[v_n]}{r}dS(\mathbf{y}). \tag{16}$$

Equation (16) is the FW-H equation in the integral form [3]. The terms in the right-hand part of Eq. (16) represent, respectively, Lighthill's quadrupole acoustic sources due to turbulence in the fluid volume [1], dipole acoustic sources on the rigid surface due to force acting between the fluid and the surface [2], and the monopole sources on the surface due to its motion [3]. Eq. (16) is reduced to Curle's equation [2] if the surface is stationary, i.e. if $v_n \equiv 0$.

It is necessary to note that, in Eq. (16) , the force acting between the fluid and the surface is determined by the tensor p_{ij} and, therefore, is due to both acoustic and viscous stresses. Analogously, the velocity v is the total velocity and contains both potential and rotational components, which are respectively related to density fluctuations and vorticity.

5. Derivation of the inhomogeneous Kirchhoff equation

The integration over the total volume V described in Section 4 is not the only way of finding the field generated by the sources determined by the right-hand side of Eq. (8). The Kirchhoff formula can also be applied to the *external* fluid volume V_2 only. In this case, the integration over V_2 takes account of the volume distribution of sources due to Lighthill's stress tensor T_{ij}. The two surface integrals in Kirchhoff formula (Eq. (13)) should now be taken over the boundary of the volume V_2, which is the *external* side of the surface S. As all field variables are continuous on the external side of S, the sources due to the field discontinuities on S are located *outside* the volume of integration V_2 and can be taken into consideration by the surface integrals in (Eq. (13)). Therefore, the Kirchhoff formula describing the solution of Eq. (8) can now be written as:

$$4\pi c_0^2 \rho'(\mathbf{x},t) = \iiint_{V_0} \frac{1}{r}\frac{\partial^2 [T_{ij}]}{\partial y_i \partial y_j} dV + c_0^2 \iint_{S_0} n_i \frac{\partial}{\partial y_i}\left(\frac{[\rho']}{r}\right) dS(\mathbf{y}) - c_0^2 \iint_{S_0} n_i \frac{1}{r}\frac{\partial [\rho']}{\partial y_i} dS(\mathbf{y}). \tag{17}$$

The derivatives and the integral in the first term in the right-hand part of Eq. (17) can be interchanged as described above. This results in the inhomogeneous Kirchhoff equation:

$$4\pi c_0^2 \rho'(\mathbf{x},t) = \frac{\partial^2}{\partial x_i \partial x_j} \iiint_{V_2} \frac{[T_{ij}]}{r} dV(\mathbf{y}) + c_0^2 \iint_{S} n_i \frac{\partial}{\partial y_i}\left(\frac{[\rho']}{r}\right) dS(\mathbf{y}) - c_0^2 \iint_{S} n_i \frac{1}{r}\frac{\partial [\rho']}{\partial y_i} dS(\mathbf{y}). \tag{18}$$

This equation should be supplemented by the boundary condition on the outer side of S (Eq. (5)). Equation (34) derived in Ref. [4] takes the following form:

$$\rho'(\mathbf{x},t) = \frac{1}{4\pi c_0^2}\frac{\partial^2}{\partial x_i \partial x_j} \iiint_{V_2} \frac{[T_{ij}]}{r} dV(\mathbf{y}) + \frac{1}{4\pi}\iint_{S}\left(\frac{1}{r}\frac{\partial [\rho']}{\partial n} + \frac{1}{r^2}\frac{\partial r}{\partial n}[\rho'] + \frac{1}{c_0 r}\frac{\partial r}{\partial n}\frac{\partial [\rho']}{\partial t}\right) dS(\mathbf{y}). \tag{19}$$

By using Eq. (12) and taking account of different directions of the normal unity vector assumed here and in Ref. [4] one can demonstrate that Eqs. (18) and (19) are identical.

6. Discussion

The argument above demonstrates that the FW-H equation in the integral form (Eq. (16)) and the inhomogeneous Kirchhoff equation (Eq. (18)), which is taken together with the boundary condition on the outer side of S (Eq. (5)), are solutions of the FW-H equation in the differential form (Eq. (8)). As both solutions satisfy the same wave equation (Eq. (8)) in the fluid volume V_2 as well as the boundary condition on the outer side of the surface S (Eq. (5)), then, according to the uniqueness theorem [15], the two solutions coincide. Therefore, it can be stated that the FW-H equation in the integral form (Eq. (16)) and the inhomogeneous Kirchhoff equation (Eq. (18)) together with the boundary condition on the outer side of S (Eq. (5)) are different forms of the same solution. As no assumptions has been made in the above argument whether the boundary is stationary or moving, it can be concluded that the inhomogeneous Kirchhoff equation (Eq. (18)) also describes the solution of the wave equation in the case of the stationary boundary.

The argument presented in this paper shows that both the FW-H equation and the inhomogeneous Kirchhoff equation can be derived from the same non-linear equation (Eq. (8)) without any additional assumptions. Therefore, it is proven here that the latter equation is fully equivalent to the former one and not just to its linearized version.

Although the two equations are equivalent, they are expressed via different variables. On the one hand, the second and the third terms in the right-hand part of the FW-H equation (Eq. (16)) contain the total components of the velocity and stresses that are related, correspondingly, to acoustic waves and turbulence present in the fluid. On the other hand, the terms of the inhomogeneous Kirchhoff equation (Eq. (18)) contain only the potential components of the velocity and stresses. As proven above, these two equations are different forms of one equation and, therefore, it can be stated that rotational components of velocity and stresses do not affect radiation of sound by flow near a solid boundary.

7. Conclusions

In this paper, it is shown that the Ffowcs Williams and Hawkings (FW-H) equation in the integral form as well as Curle's equation for a stationary boundary and the inhomogeneous Kirchhoff equation derived previously by the present author and his co-author are different forms of one equation. Therefore, it has been demonstrated here that the equation proposed in [4] is valid and fully equivalent to the FW-H equation and not just to the linearized version of it. This novel equation may lead to a refined framework for estimation of acoustic noise generated by flow around elastic and rigid structures. It is also shown that, whereas the surface integrals in the FW-H equation contain the total velocity and stress, only potential components of the velocity and stress, which are related to density fluctuations in the fluid, are the sources of acoustic waves at a rigid surface immersed into fluid flow.

References

[1] Lighthill, M.J. "On sound generated aerodynamically. I. General theory", *Proceedings of the Royal Society A*, **221**, 564-587, (1952).

[2] Curle, N. "The influence of solid boundaries upon aerodynamic Sound", *Proceedings of the Royal Society A*, **231**, 505-514, (1955).

[3] Ffowcs Williams J.E. and Hawkings, D.L. "Sound generation by turbulence and surfaces in arbitrary motion", *Philosophical Transactions of the Royal Society of London A*, **264,** 321-342, (1969).

[4] Zinoviev A. and Bies, D.A. "On acoustic radiation by a rigid object in a fluid flow", *Journal of Sound and Vibration*, **269**, 535-548, (2004).

[5] Farassat, F. "Comments on the paper by Zinoviev and Bies "On acoustic radiation by a rigid object in a fluid flow", *Journal of Sound and Vibration*, **281**, 1217-1223, (2005).

[6] Farassat, F. and Myers, M.K. "Further comments on the paper by Zinoviev and Bies, "On acoustic radiation by a rigid object in a fluid flow", *Journal of Sound and Vibration*, **290**, 321-342, (2006).

[7] Zinoviev A. and Bies, D.A. "Author's reply to: F. Farassat, comments on the paper by Zinoviev and Bies "On acoustic radiation by a rigid object in a fluid flow", *Journal of Sound and Vibration*, **281**, 1224-1237, (2005).

[8] Zinoviev, A. and Bies, D.A. "Authors' reply", *Journal of Sound and Vibration*, **290**, 548-554, (2006).

[9] Zinoviev A. and Bies, D.A. "Corrigendum to "On acoustic radiation by a rigid object in a fluid flow", *Journal of Sound and Vibration*, **333**, 5653, (2014).

[10] Zinoviev, A. “On conditions of equivalence between Curle’s and non-uniform Kirchhoff equations of aeroacoustics”, *Proceedings of the ICA 2010*, Sydney, Australia, 23-27 August 2010.

[11] Zinoviev, A. “On the equivalence of the Ffowcs Williams - Hawkings and the non-uniform Kirchhoff equations of aeroacoustics”, *Proceedings of the ACOUSTICS 2011*, Gold Coast, Australia, 2-4 November 2011.

[12] Stratton, J.A. *Electromagnetic theory*, McGraw Hill, New York, 1941.

[13] Korn, G.A. and Korn, T.M. *Mathematical handbook for scientists and engineers*, 2nd edition, McGraw-Hill, New York, 1971.

[14] Ikeda, T., Enomoto, S. and Yamamoto, K. “Quadrupole effects in the Ffowcs Williams Hawkings equation using permeable control surface”, *Proceedings of the 33rd AIAA Aeroacoustics Conference*, Colorado Springs, 4-6 June 2012.

[15] Carrier, G.F. and Pearson, C.E. *Partial Differential Equations: Theory and Technique*, Academic Press, London, 1976.

[16] Bracewell, R.N. *The Fourier Transform and its applications*, 3rd edition, McGraw-Hill Higher Education, New York, 2000.